Amy Johnson: Anyways, welcome to the first of the 2019 Dresden Lectures. I'm Amy Johnson. I'm the Chair of the Department of Mathematics and Statistics. I would like to just explain what the Dresden Lectures are all about before I have my colleague introduce our speaker. Arnold Dresden, here he is. So, Arnold Dresden was a professor here at Swarthmore College, from 1927 until he retired in the 1950s. He first came to the States from Amsterdam, in 1903, when he was about the same age as the juniors and seniors here.

Amy Johnson: I was told he worked at Marshall Fields Wholesale Warehouse for $10.00 a week.And that's how he saved up the money so he could go to the University of Chicago for his PhD, with a dissertation titled, The Second Derivatives of the Extremal-Integral. He then was a professor at the University of Wisconsin before coming to Swarthmore, except for sabbatical when he worked for the Red Cross during the war. So that tells you a little bit about Arnold Dresden, but doesn't tell you why we have these talks.

Amy Johnson: At Swarthmore, Dresden was a beloved professor. He was known to be very friendly, he held weekly chamber music sessions, and he was passionate about mathematics. To him, mathematics represented a philosophy of life and a discipline of the mind, which was an int in itself. He had infinite faith in his student's abilities, and his students worked very hard because they just didn't want to disappoint him. It was his students that provided the donations that then endowed this lecture series, which lets us have a famous person come each year to come and give some talks. So, with that, I'm going to have my colleague, Steve Wang, introduce our speaker.

Steve Wang: Okay, so Professor Francis Su is coming to us today from the Harvey Mudd College, which is one of the Claremont colleges. He is the Benediktsson-Karwa professor of mathematics. He did his undergrad in math at the University of Texas, and then did his PhD at Harvard in math, with Persi Diaconis, known as mathematician slash magician. In addition to teaching at Harvey Mudd, he's also taught at Cornel and Cal Tech, and has won pretty much every teaching award there is to win.

Steve Wang: In addition, he was president of the Mathematical Association of America for many years. He is a great researcher, having written many papers with undergrad student co-authors, but also has a passion for popularizing math and has run a website of math fun facts. I decided to check that out last night, and easily spent a half an hour poking around. His work on fair division vision has been covered by the New York Times.

Steve Wang: Fair division is the topic of how to split a bill. For instance dinner tonight, we'll have him calculate who should pay what amount. Or, if you want split rent with your roommate and so on, so that's been covered in the New York Times. His hobbies include gardening, photography, theology, and songwriting. We're delighted to have him with us today, so please welcome Professor Francis Su.

Francis Su: Thanks a lot. Thank you very much. Thanks for the generous introduction. It's a real pleasure to be here at Swarthmore. Is it Swathmore or Swarthmore? After hearing so much about this school for many years... Also, a real pleasure to visit with my grad school friend, who's now a professor here, Professor Catherine Crouch. So yes, your professor were once students before, and visit with many friends here in the math department as well.

Francis Su: I want to start off this talk by just asking the question, how is math connected to what it means to be human? Think about what it means to be a human being. Do you think of math as being connected to qualities of being human? Why does the subject of math produce joy for some people and anxiety for others? There's a question for us to ponder. Think about something you're passionate about. Whether that's math, or whether that's a sport, or whether that's music. Why are you passionate about those things?

Francis Su: Maybe take a moment and share one thing with your neighbor that you're passionate about, and one reason why. Very quickly. If you think about why you're passionate about the things you're passionate about, I would say it's because whatever you're passionate about, you're drawn to because of basic human desires. And maybe math would be more attractive to many of us, if we could see how math connects to our basic human desires. I love this quote by Simone Weil, "Every being cries out silently, to be read differently."

Francis Su: I have a friend who's a prison inmate in a Florida prison. His name is Christopher, he's been in trouble with the law since he was 14. He had an addiction of hard drugs throughout his teenage years. He was involved in a string of armed robberies and that led to him landing in prison at age 19, with a 32 year sentence and no chance of parole. When you think about somebody like Christopher... When you think about who does mathematics, would you think of somebody like Christopher? There's a question.

Francis Su: Yet, he wrote me this letter after seven years in prison. He said this, "I've always had a proclivity for mathematics, but being in a very early stage of youth, but also living in some adverse circumstances, I never came to understand the true meaning and benefit of an education. Over the last three years, I have purchased and studied a multitude of books to give me profound understanding of Algebra I, Algebra II, College Algebra, geometry, trigonometry, Calculus I, and Calculus II."

Francis Su: Christopher was now writing me for help in furthering his mathematics education, so I ask again. When you think of who does mathematics, would you think of Christopher? And yet, Christopher's been on a journey. He's exploring math with fresh eyes, beginning to see that it's different from the dry and uninspired form of math that he had seen before. He's growing in his knowledge and love for the subject, and his insights have inspired me, a professional mathematics, to believe even more fully that math has something to offer everyone.

Francis Su: "Every being cries out silently, to be read differently." Simone Weil is a well known, French, religious, mystic and a revered philosopher. She's probably less well known as the younger sister of Andre Weil, one of histories most famous number pairists. And for Simone, to read someone means to interpret, or make a judgment about them. Every being cries out silently, to be judged differently. I sometimes wonder if Simone were crying out about herself, because she too loved and participated in mathematics.

Francis Su: She was always comparing herself to her older brother. And she writes this, "At 14, I fell into one those fits of bottomless despair that come with adolescence, and I seriously thought of dying because of the mediocrity of my natural faculties. The exceptional gifts of my brother that childhood and youth, comparable to those of Pascal, brought my own inferiority home to me. I did not mind having no visible successes, but what did bereave me was the idea of being excluded from that transcendent kingdom to which only the truly brave have access, and wherein truth abides. And I preferred to die, rather than live without that truth."

Francis Su: Now we know Simone loved mathematics, because she used mathematical examples throughout her philosophical writing. You'll find her in photos of Bourbaki, a group of French reformist mathematicians that included her brother. You'll see Simone here on the left, looks like she's pouring over some notes, and Andre is the one waving the bell. I often wonder what her relationship to mathematics would be like if she weren't always in Andre's shadow. "Every being cries out silently, to be read differently."

Francis Su: Now you might look at me, a math professor, and think that my relationship to math has always been solid. Maybe you think I've never struggled. Like Christopher, I gravitated towards math as a youth, but I grew up in a small town in south Texas. Yes that's me, with hair. And there were limited opportunities in my small, rural town. Most of my high school friends didn't go to college. I had supportive teachers throughout high school and at university, and I managed to get admitted to Harvard for my PhD. But I felt out of place there, because unlike many of my peers, I did not come from an Ivy League school. And unlike many of them, I did not have a full slate of graduate courses when I entered.

Francis Su: I felt like Simone Weil, standing next to future Andres, wondering if I could flourish if I were not like them. I was told by one professor that I didn't belong in graduate school, and that caused me to wonder, "Why in the world am I doing mathematics?" That's a question I want all of us to think about today. Why do mathematics? Why is Christopher sitting in a prison cell studying calculus that he'll not use as a free man? At least for another 20 years. What does math have to offer him? Why was Simone captivated by transcendent mathematical truths? Why should anybody persist in doing math, or seeing yourself as a mathematical person, when others are telling her in subtle, and maybe not so subtle, ways that she doesn't belong?

Francis Su: Society is also asking what its relationship to mathematics should be. Is math only a tool to make you, "college and career ready," Or is math unnecessary for most of us, and only for an elite few? We've all maybe heard this question, or even asked this question. Why study math if you'll never use what you're learning? These questions often arise because math is often just seen as a means to an end. A tool that you're going to use later, rather than a pursuit that serves you well, right now.

Francis Su: So when people ask, "When will I ever use this?" What they're really asking is, "When will I ever value this?" They equate value with utility, because they haven't been told that they can value anything more. And yet, in a Florida prison, Christopher isn't asking this question, "When will I ever use this?" He's pouring over textbooks, learning math, in hopeful anticipation of its own intrinsic rewards. So, if you ask me, "Why do mathematics?" I would say this, because mathematics helps people flourish.

Francis Su: Mathematics is for human flourishing. And when I say human flourishing, what I'm referring to is a wholeness. A wholeness of being and of doing, of realizing one's potential and helping others to do the same, of acting with honor, and treating others with dignity. Of living with integrity, even in challenging circumstances. It's not the same as happiness and it's not just a frame of mind. Another way to think about human flourishing is, the well lived life. And this is the way that some of the ancient Greek philosophers thought about this. And the ancient Greeks have a word for human flourishing, eudaimonia, which they viewed as the overarching good in life. The good composed of all goods in life. The well lived life.

Francis Su: There's also a similar word in Hebrew, shalom, which is often used as a greeting. The word is sometimes translated, peace, but it has a far richer meaning. To say shalom to somebody, is to wish that they would flourish and live well. And Arabic has a similar word, salaam. And a basic question taken up by Aristotle, and philosophers throughout the ages, is, how do you achieve human flourishing? What is the well lived life? Aristotle would say, the flourishing comes about through the exercise of virtue.

Francis Su: And the way I think about virtue is, excellence of character that leads to excellence of conduct. It includes more than just moral virtue. For instance, courage and wisdom are also virtues. And so what I hope to convince you of today, is that the practice of mathematics cultivates virtues that help people flourish. These virtues serve you well, no matter where your life takes you or what profession you choose. The movement towards virtue happens through basic human desires. So today I want to talk about seven human desires that all of us share.

Francis Su: The first of these is exploration. When I was a child I loved the stars, and in my small rural town, far from any big city, I could see a lot of stars. It's very different now. I live in L.A. and you can't even see a star in the sky. Most people who grew up in L.A. are probably surprised that stars even exist. I remember begging my parents for a telescope, but we didn't have the money, so I just devoured books on astronomy and I dreamed about space. My imagination was stoked at the time by the journey of the Voyager probes, this was in the 1970s.

Francis Su: The Voyager probes through the solar system, and I saw pictures like these printed in the papers. You see, exploration is a deep, human desire, and it's a mark of human flourishing. I could explore these worlds, even from 900 million miles away. Math exploration is very much like space exploration, but a different kind of space. It's a space of ideas. You don't know what you'll find when you start. You send out probes to test theories. You're captivated by mystery. You're motivated by questions. You aren't phased when you encounter setbacks, but you press on and you make discoveries from a distance, because the ideas themselves are not physical. You access this space through reason.

Francis Su: Exploration is the heart of what it means to do mathematics, and you don't need a lot of resources, except your mind, to be a mathematical explorer. As a result, you can embark on an adventure from anywhere. A prison, a small rural town, or a far flung corner of the universe. An exploration cultivates in us, virtues like imagination and creativity. In order to solve problems, you have to ideate new possibilities. Mathematical explorers like to imagine new ways of visualizing patterns, and they begin to find creative ways to envision the hidden structures that underlie everything we do.

Francis Su: It's at the heart of many of the current, technological innovations that we see, that we use on a daily basis. New ways to envision hidden structure. That's really what mathematics is. Exploration also cultivates, in us, the virtue of an expectation of enchantment. Explorers are excited by the thrill of finding the unexpected, especially things weird and wonderful. It's why hikes through unfamiliar terrain beckon to us. It's why unexplored caves entice us. It's why the strange creatures of the deep sea ocean floor fascinate us. "What else might be lurking down there," we wonder.

Francis Su: In math, it's the same way. When we take a fractal Menger sponge, which I know all of us have in our houses, with self-similar squares and cubes all over. And then we slice is diagonally, and then find a bunch of hexagonal scars... Yeah, go ahead and say...

Audience: Ooh.

Francis Su: We're enchanted. We're amazed. We ask that fundamental, human question. Why? What's going on? This make math exciting to learn, because we learn to expect enchantment around every corner, so we keep coming back for more. What would it be like if, as we learn math, we opened ourselves up to enchantment? If you think math is about memorizing things and not making sense of things, then of course you're going to think it's dull and boring. But for Christopher, math isn't just a tool, it's hidden treasure.

Francis Su: In every opportunity, we need to counter the idea math is memorization and replace it with the idea that math is exploration. A math memorizer doesn't know how to react in unfamiliar situations. Wow. A mattress, floor, can flexibly adapt to changing conditions. We can all be math explorers, because it is a deep human desire.

Francis Su: There's a second human desire, play. Think about various ways that you play. Sports play, musical play, beach play, board games, poetic word play. One of the first activities we engage in, as babies, is play. Now what makes play, play? Well, play is a little bit hard to define, but we could probably think of a few qualities that characterize it. For instance, play ought to be what?

Audience: Fun.

Francis Su: Otherwise it wouldn't be play. But there's also many other qualities, play usually involves some structure, like the rules of a game or musical chords, but there's often lots of freedom within that structure. That freedom leads to exploration of some sort. Like in jazz it's called improvisation. In sport, it's all the different ways that the game can unfold each time it's played. Play is often whimsical, with no great stake in the outcome. For instance, we might care if we win a game today, but next week it won't matter.

Francis Su: The investigation often leads to some sort of surprise, like a discovery, or a satisfying pun in word play, or a thrilling end to a football game. And of course animals play too, but what characterizes human play is the enlarged role of the mind and the imagination. So think about any game you play, like the game SET. How many people here have played the game SET? It's a very mathematical game and it's a game that kids actually play a lot better than adults, turns out, but it involves mathematical thinking. There's interplay between structure and freedom in any game. There's no great stake in the outcome, but there's an investigation that can lead to the delight of finding a set of matched cards.

Francis Su: Mathematics makes the mind its playground. We play with patterns, we play with ideas, we explore what's true, and we enjoy the surprises along the way. There's even a whole area in mathematics, it's actually a field of mathematics known as recreational mathematics. Is there another field that has a recreational sub-discipline? My chemist friend said, "Well there is recreational chemistry." I guess you're right. Mathematics play builds, in us, virtue. Math ables us to flourish many area of our lives. For instance, math play builds hopefulness and concentration. When you sit with a puzzle long enough, you are exercising hope that you'll eventually solve it.

Francis Su: Math play builds the ability to change perspective. Just like in game play, when you analyze a situation from multiple points of view, yours and your opponents, and you imagine possibilities. Math play builds up perseverance, just as weekly soccer practices build up muscles that make us stronger for the next game. Weekly math investigations, problem sets, make us more fit to solve that next problem. Yes? Even if we don't solve the current problem.

Francis Su: Play is part of human flourishing and you cannot flourish without play. So, as we learn math, let's make room for questions, investigations, surprise, imagination. Let's de-emphasize the role of grades in our minds, it's just a measure of progress, not a measure of promise. Let's focus on the process of doing math, not just focusing on the outcome of a calculation. Let's make math a playful sport, not a performance sport.

Francis Su: There's a third basic human desire, beauty. I love this quote by Sofia Kovalevskaya, one of the great mathematicians of the 19th century, "It's impossible to be a mathematician without being a poet in soul." Who among us doesn't enjoy beauty or beautiful things? A beautiful sunset, a sublime sonata, profound poem, an illuminating idea. You can experience mathematical beauty, even if you don't know much math. You might first see it in striking patterns, governed by mathematical laws, that are as beautiful as a sunset, the turbulence under an airplane wing, fractal patterns in cauliflower.

Francis Su: This is a picture I took. When I was in England I was in the supermarket and I was like, "Oh, my gosh. That's a fractal cauliflower." How many people have seen this? A Romanesco cauliflower. Ripples in sand... Mathematicians study all these things for the patterns that are in them, but you don't need to understand or know advanced math to simply ask the question, "Why?" And why, is the most fundamental mathematical question. When you begin to study math, you begin to see a more sublime kind of beauty. The beauty of a formula that seems to connect surprisingly unrelated things. Or the simplicity, regularity, and order of the laws and universe. These are called beautiful. They feel transcendent, because they say something about the nature of the universe.

Francis Su: Many people have asked this philosophical question, "Why should math be as powerful as it is, to explain the world?" You could imagine a whole world where math is just a bunch of little things you do on a piece of paper, that has no connection to reality. Nobel Prize winning physicist, Eugene Wigner, asked this question. And he said this, "The miracle of the appropriateness of the language of math, to the formulation of the laws of physics, is a wonderful gift which we neither understand nor deserve." Whoa.

Francis Su: Einstein asked, "How can it be that math, after all being a product of human thought and independent of experience, is so admirably adapted to the objects of reality?" Mathematicians try to understand why something is true by producing proofs, but we're often not satisfied with just any old proof of a theorem. We often look for the best proofs, the simplest, or the most pleasing arguments. Mathematicians have a special word for this. We say a proof is? Elegant.

Francis Su: Paul Erdos, a prolific mathematician, often spoke of the book that God keeps, in which all the most elegant proofs are kept. In pursuing math for it's beauty, beauty cultivates in us, virtues. Like the virtue of reflection when you contemplate beautiful ideas, just like you contemplate beautiful forests. It leads to joyful gratitude and transcendent awe. When we experience profound beauty from seeing a beautiful idea everywhere, we build, in us, habits of generalization, because we begin to expect overarching patterns everywhere.

Francis Su: Let's dwell on that for a moment. You are being trained at one of the best colleges in the nation. And it is, probably, now a common experience for you to actually think this world can be understood. Right? You begin to see and expect patterns everywhere, but not everybody has that hope. Not everybody has that hope. When I learn a new theorem now, I often ask, "What gives this underlying theorem its power? What's the underlying principle? How might it apply to a more general situation?"

Francis Su: And such habits carry over to all the areas of my life. When I'm cooking a stir fry and I learn a new recipe, I often ask, "What general principles does this recipe teach me?" I don't want to sit there and memorize every recipe. I just want to remember the principles, add garlic and chop onions first. Put in basil at the end or it's going to lose its color. This habit of looking for general cooking principles, or mathematical principles, allows you to improvise new delights. And abound the stream of flourishing, we have to help others see its beauty. If you want to see some things that I think are beautiful, you might enjoy this collection of math fun facts that I've made. It's on our website, just Google math fun facts.

Francis Su: I've often used these little morsels as ways of showing some of my students that math is beautiful. We have to make reflection a part of our mathematical experiences. What beautiful patterns do you see in the worlds around you? What hidden structures do you think might underlie what you see? We have to motivate beauty in diverse ways, to mirror all the ways that people might come to math. Through art, through music, through expressions and culture, through patterns, through rigorous arguments. Through the elements of simple but profound ideas through applications.

Francis Su: Here's another human desire, the desire for justice. This is the full quote by Simone Weil, "Justice. To be ever ready to admit that another person is perhaps something completely different from what we read in him. Every being cries out silently, to be read differently." Now, my favorite Chinese restaurant makes authentic Chinese cuisine, just like my mom used to make. When you order an entrée, they bring you a little appetizer and a dessert as well. It's a bargain, so I don't complain that the appetizer and they dessert, Jello, are not themselves authentic. I don't complain, because they're free.

Francis Su: But one day I went there with a Chinese speaking friend and when the appetizer came it wasn't little crunch noodles, but delectable pickled cucumbers, yet my friend had made no special request. And when the dessert came, it was red bean soup just like my mom used to make. How come I hadn't gotten this before? I began to see a pattern. When I came to this restaurant with non-Asian friends, I would get the crunchies and the Jello. But when I came with my Asian friends, I got the good stuff without even asking. Then I noticed that my Chines friends were also offered a completely different menu. A secret menu with more authentic dishes.

Francis Su: I looked around the restaurant and I beheld a bizarre sight. People side by side, in the same space, having very different experiences. Non-Asians ordering from a standard menu and getting Jello, Asians were in on a secret menu and enjoying red bean soup. And even when I asked for the secret menu, now that I knew about it, I was often discouraged from it. I told, "No. No, no, no, no. You won't like the stuff on that menu." You see, because even though I'm of Chinese descent, the fact that I didn't speak any Chinese, meant that they assumed that I wouldn't be interested in authentic Chinese food.

Francis Su: Now why do I tell the story? I would say mathematical spaces in the home and in the classroom, can be often like this restaurant. Who do we allow a peek at the secret mathematical menu? With whom do we share our mathematical delights? Puzzles or games or toys. Who do we let into our information circle about mathematics? Whom do we shepherd towards taking more math, and whom do we discourage? What conscious or unconscious assumptions are we making? You see, justice means treating people fairly and it's required for human flourishing. We flourish, we experience shalom or salaam, when we are treated justly and when we treat others justly.

Francis Su: And Simone Weil recognized that correcting injustice must involve changing how we view others. If you believe math is for human flourishing, you'll see, if you look around the demographics of who does math and science in America, that we've kept whole segments of people away from the flourishing available in mathematics. Either by lack of access or explicit messages of disapproval, and we often don't realize we're doing it. These days I'm used to being in math conferences and seeing a sea of white faces, so even I was a little bit surprised when I was elected president of the Mathematical Association of America. That a prominent blogger on race issues for Asians wrote a blog post about it, and his name is Angry Asian Man.

Francis Su: Angry Asian Man looked at photos of past MA presidents on our website, and given how many Asians he expected to be in, they aptly noted they were all white except for me, and wrote a sarcastic post titled, Finally. An Asian Guy Who's Good At Math. By the way, Arnold Dresden is a past MA president and should... Look, right there. By the way, Dresden's great, great, nephew is in the audience and he's right there. He's a math professor at Washington and Lee. What is Angry Asian Man saying here? His sarcasm is directed at the idea that he expected to be... Because there's lots of Asians doing math, he expected to see lots of Asians in leadership.

Francis Su: The dominant narratives is, often, that Asians can't be leaders. And so what other assumptions do we make about other people groups? You see, to seek justice in mathematics builds many virtues. Among them, a concern for the marginalized and a willingness to challenge the status quo. When we begin to see how our experiences are different from our neighbors, we will want the secret menu for everyone. Isn't that our basic human desire, freedom? Maybe you don't associate the idea of freedom with mathematics, but there are many freedoms we take for granted. The freedom to imagine, the freedom to explore, the freedom of understanding. They're not there for all of us.

Francis Su: I've had a rich correspondence with my friend Chris in prison, about freedom, and he's made some astute observations. He said this, "At a most basic level, freedom is the ability to act without fear." But as he came to understand, from playing chess... By the way, he's an amazing chess player. I played him twice now and lost both times. Freedom is also knowing about all the options available to you. Another player can dominate you and control you by limiting your options on a chess board. This is what he says, "It's a sign of a skilled chess player that he or she can play well out of any position or circumstance on the board. The person who is unaware of his or her options is like a player who's in a bad position. Because even if there are fruitful paths available to you, that you are not aware of, they may as well no exist."

Francis Su: This is like if a player finds his self with two bishops against a lone king, but doesn't know the two bishops can check mate, he or she will call a stalemate. But when this person is shown, educated, that two bishops can checkmate, they will always recognize the situation as a win. And this is, in my opinion, the main bridge of education. To take people to a place where they can recognize the pathways to success. By climbing our visions of ourselves, education allows us to transcend ourselves and thereby help others to do the same.

Francis Su: You see, if you have pursued the freedom that mathematics brings, and understanding things, you build resourcefulness. You begin to have hope that problems can indeed be solved, and many people don't have that hope. We can help people have the hope that problems can be solved. You become fearless in asking questions. You become an independent thinker and pursuing mathematics is a way to become a freer human being.

Francis Su: Another basic human desire is community. This is one of my favorite quotes about teaching, from Parker Palmer. It says, "To teach is to create a space where the community of truth is practiced." You see, teaching is not just a one way transmission of information. Good teaching, good learning, takes place in a community. Ricardo was a New York City native, a son of immigrants in a working class neighborhood. His father never finished high school and mother didn't make it past the eight grade. He had shown an aptitude for math at an early age, but he lacked a mentor to guide him towards the [inaudible 00:38:11] tracks, so he ended up in another career.

Francis Su: But now he's taken the brave step of going back to school at age 40, in a program for non-traditional students. He wrote me a letter a few years ago, after reading a version of my talk online. Here's what he said, "While the rigor and the pressure of being in a really tough academic environment had been almost impossibly difficult, especially after being out of practice for so long, is the entrenched feeling that I don't belong in these math and CS classes that has sometimes been the most harmful."

Francis Su: "These feelings are probably most tied to my early life, and the fact that any dreams I might have had then were in discord with the cold realities of my neighborhood and life. That I didn't have any mentor then to disabuse me of that great untruth. In that distorted reality, I'm not supposed to be here. Runs is an infinite loop process, even in unseen ways. It is a constant struggle."

Francis Su: You can't flourish as a human being without a supportive community. A community of people who open up with each other and share life together, who share joy and sorrows together, who even share math together. A community helps us normalize struggle and helps us realize, "I'm not alone in my struggle." Doing math well means making space for community, but we have to be mindful that math communities are often too focused on achievement, often a narrow, one-dimensional kind of achievement. Hierarchies begin to get reinforced when we rank people according to a singular ability, and this works against community.

Francis Su: For instance, we do this when we think there's only one way to be successful in math. In grade school, people looked at speed as a measure of math ability. But those of us who are professional mathematicians know that speed isn't limitless pass for math ability. In fact, it's often said that math isn't about doing things quick, math is about actually learning ideas slowly, letting them sink in, beginning to understand them at a deeper level. And we have to realize, there are many ways that we can show growth in mathematics.

Francis Su: In high school, this misconception about math being successful only one way, looks like rushing to take calculus. And why do you do it? Because our educational system says if you take calculus, you'll get into a good college like Swarthmore. But in reality, there are many more ways to show evidence in growth in mathematics. In college, this misconception looks like starting in advanced place class. Thinking, "Oh, I'm only going to be good enough to study math. I placed out of calculus."

Francis Su: For a professional mathematicians it looks like thinking that the only important dimension to being a mathematical scholar, is the number of papers that you publish. And we forget there are many more ways of being mathematical. Too often we think of math as a pole in the ground and there's only one way for the vine to grow up the pole. In reality math is like a trellis, and as a vine you can find your way up the trellis of multiple places where it meets the ground, and you can grow in multiple directions. So we have to fight this one dimensional view of mathematics. In the classroom or the home, many of you will become parents someday, you'll go coach somebody in thinking about math.

Francis Su: You can praise people for showing growth in the virtues that are brought about by doing mathematics. Remind them that this is part of doing mathematics too. Persistence, curiosity, imagination, habits of generalization, a disposition to beauty, these are all ways to evidence growth in mathematics. As a community, we can help one another see each other's humanity. This is a huge part of doing mathematics. By practicing mathematics in this way, we build virtues like hospitality and welcoming new people to math. Vulnerability in sharing our struggles, and normalizing that struggling is actually an important part of what it means to do math.

Francis Su: If you're not struggling in some math class, you aren't really, fully living a mathematical experience. Struggle is such an important art of doing math and being able to look to a past experience when you struggles, and you've overcome that struggle, is part of what it means to understand the thrill and the joy of doing mathematics. We develop a disposition to affirm others, and we build our ability to give attention to people in a deep way when we practice mathematics through the lens of community. You can do this in your classes, in the math classes you're taking. You can praise others for asking the question that you were afraid to ask, because you didn't want people thinking that you didn't know something. And I'd done that all the time where I'm like, "Oh, I want ask this question, but I don't want to be seen as not knowing anything." So I don't raise my hand.

Francis Su: I got to grad school and I realized I wasn't going to learn anything if I didn't ask questions. I started asking questions and looking like the dumb one. And guess what? People started thanking me afterwards for asking those questions. So if you're not brave enough to ask questions, that's okay, but thank the person who asked the question that you wish you [crosstalk 00:44:25]. All right? Build community. This should be part of doing math too. You see, don't let people tell you that math is just logic, heartless, a bunch of rules to follow. Who would want to learn that or teach that? That's not mathematics.

Francis Su: You can't separate the true practice of math and what it means to be human, because we are not just mathematical machines. We live, we breathe, we feel, we bleed. If you want to learn math well, you have to see how math connects to the things that you long for, the things that matter, the things that drive all of us to do any of the things we're passionate about. Desire for exploration, for play, for truth, for beauty, for justice, for freedom, for community. So let me challenge each of us today, even myself. Believe that you, and everyone in your life, can flourish in mathematics.

Francis Su: Who will you read differently? This is a challenge for me too, because I've fallen short of the set deal. I've written off kids that I didn't think had the potential, because I didn't have the imagination that I needed for the sake of responsibility teaching. You might know a Christopher in your life, who's fallen in with drugs and the wrong crowd. He never seemed interested in math, maybe he even seems lazy. If you knew what he was going through, you might read him differently.

Francis Su: Six years after he wrote me that first letter from prison, he's helping other inmates to learn math to get their GEDs. He's using his meager income he makes, to buy math books. He's studying topology and advanced analysis now. And he says this, "I'm studying anywhere between three to five hours, Monday through Friday, and two plus hours Saturday and Sunday, depending on how I'm feeling. It's harder to study and read here, because you're not in a traditional cell with a cell door you can be enclosed in. Everything's open, we live in an open top, eight by ten foot cube, with walls that extend six feet off the ground and a three foot gap, on the eight foot side, to act as your door."

Francis Su: "Also, I'm in a room without a desk, so I had to take two chairs to do studying in. But hopefully that will change soon, because I'm in the works to try to move to a cube with a desk. But I can't complain too much, I grab my earplugs, my two chairs, and I go to work." When I think about all the petty little things that I complain about in my life, and he's just trying to get a desk so he can study mathematics. See, nobody would call him lazy or disinterested in math. Would have I had the imagination to see a math future for him ten years ago? Believe that you, and everyone in your life, can flourish in mathematics.

Francis Su: What I'm asking you to do is something you already know is at the center of every virtue and everything meaningful in our lives. I'm asking you to love, so there's the picture. There's the picture. Love is the greatest human desire, and to love and be loved is the supreme mark of human flourishing, for it serves all the other desires. For play, for truth, for beauty, for justice, for freedom, for community, and it's served by that. Love is the source, and end, of all virtue. "Every being cries out silently, to be read differently." Every being cries out silently to be loved.

Francis Su: Christopher in prison wasn't just looking for mathematical advice, he was looking for connection. For somebody to reach out to him and his mathematical space, and say, "I see you, and I share the same passions that you do for mathematics. You belong." And when I was in the depths of despair in graduate school, struggling against professors who thought I would never succeed and I was ready to give up, one professor reached out to me. He said, "I would rather you work with me than quit." So I stand before you to ask you, who will you love? Who will you read differently?

Francis Su: I'll end with a few reflections. Simone Weil, after struggling with her own insecurity in mathematics, saw that there was a path to virtue through her struggle. She wrote this, "The love of our neighbor in all its fullness simply means being able to say him, 'What are you going through?'" It's a recognition that the sufferer exists. Not only is a unit in a collections, or a specimen from a social category labeled unfortunate, but is a man exactly like us who is one day stamped with a special mark by affliction. But this reason is enough, but it is indispensable to know how to look at him in a certain way."

Francis Su: "This way of looking is, first of all, attentive. The soul empties itself of all its own contents in order to receive into itself, the being it is looking at just as he is in all his truth. And only he who is capable of attention can do this. So it comes about, paradoxical as it may seem, a Latin prose or geometry problem, even though they are done wrong, may be of great service one day, provided we devote the right kind of effort to them. Should the occasion arise, they can one day make us better able to give someone in affliction exactly the help required to save him, in a supreme moment of his need."You see, she had found a path through struggle, to virtue. And she understood that mathematics is for giving flourishing.

Francis Su: This is from Ricardo, our 40 year old friend who returned to school. I had a chance to meet him in New York when I was there recently. He says this, "I'm in classes with 20 year olds and I'm having the time of my life. This learning has unlocked much I didn't know existed in myself. Since I've been back I've struggled with math, calculus has really beat me up, but after a 20 year break from it, I'm finding it's harder to relearn. Finding it impossible to imagine I was ever really good at this. But even in the pain of failure, of trying to reshape my brain to comprehend, I feel more alive than I ever have before." He understands mathematics is for giving flourishing.

Francis Su: And this final letter is from Max, who wrote me after reading a version of my talk online. And he said this, "I just finished reading your wonderful article and felt compelled to reach out with a small personal story. When I was in second grade, I struggled with subtraction. I asked my teacher for help. She snapped, told me some mean spirited, quibble note, 'You need to go figure this out, because it isn't all right.' I returned to my desk feeling like the biggest idiot. I barely ever asked for math help after that, and struggled for mediocre grades until college."

Francis Su: "But in college, I fell in love with an aerospace engineering major. And her deep focus [crosstalk 00:51:42]. Half of you do too. And at the same time, I discovered a passion for economics. And through that, [inaudible 00:51:52] abilities elegantly explained complex [inaudible 00:51:54]. I only have a undergrad degree, but I manage to work and apply that ever since graduating, today do time series analysis in health care. If only I could tell eight year old me of this trajectory."

Francis Su: Discovering the beautiful intersection of math and the humanities will always have a very special place in my heart, and I love to share it with others. The path I've been on has shaped my perspective that anyone, regardless of gender, ability, race, or otherwise, can be part of this wonderful thing. You see, math can be so much more than we make of it when we see how it's connected to the things we long for, to our basic human desires. And if that's the case then everyone really is a math person, because your basic human desires reveal your mathematical nature and you only need to awaken it.

Francis Su: And what love is, to love another human being, needs to see each person as worthy of the beauty and the joy that comes with understanding, to see each person like that. To be able to look at each person and envision a future, even a mathematical future, that they can't see for themselves. Thank you very much.