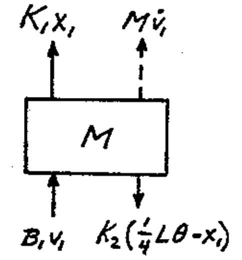
E12 ASSIGNMENT 6 SOLUTIONS

5.13

We do not include the gravitational force Mg in the free-body diagram for the mass, because displacements are measured from the static equilibrium condition caused by gravity. Note that the free-body diagram for the lever is labeled with torques. From (5), we obtain

$$J_A = \frac{M}{3L}\left[\left(\frac{L}{4}\right)^3 + \left(\frac{3L}{4}\right)^3\right] = \frac{7ML^2}{48}$$

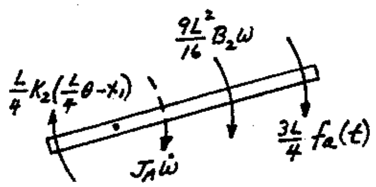
In drawing the diagrams, we used $x_2 = (L/4)\theta$, $x_3 = (3L/4)\theta$, and $v_3 = (3L/4)\omega$. Summing the forces on the mass and summing the torques about the lever's pivot point, we have



$$\begin{aligned} M\dot{v}_1 + B_1v_1 + (K_1 + K_2)x_1 - \frac{LK_2}{4}\theta &= 0 \\ \frac{7ML^2}{48}\dot{\omega} + \frac{9L^2B_2}{16}\omega + \frac{L^2K_2}{16}\theta - \frac{LK_2}{4}x_1 + \frac{3L}{4}f_a(t) &= 0 \end{aligned}$$

We select x_1, v_1, θ, and ω as the state variables and write

$$\begin{aligned} \dot{x} &= v \\ \dot{v} &= \frac{1}{M}\left[-(K_1 + K_2)x_1 - B_1v_1 + \frac{LK_2}{4}\theta\right] \\ \dot{\theta} &= \omega \\ \dot{\omega} &= \frac{3}{7ML}[4K_2x_1 - LK_2\theta - 9LB_2\omega - 12f_a(t)] \end{aligned}$$



The output equation is

$$f_{K_2} = -K_2x + \frac{LK_2}{4}\theta$$

6.9

Let e_A denote the node voltage at the top of the inductor, and i_L the current down through the inductor. Applying Kirchhoff's current law to nodes A and O gives

$$\begin{aligned} -i_i(t) + i_L(0) + \frac{1}{8}\int_0^t e_A d\lambda + \frac{1}{4}(e_A - e_o) &= 0 \qquad \text{(A)} \\ \frac{1}{4}(e_o - e_A) + \frac{1}{2}\dot{e}_o + \frac{1}{4}e_o &= 0 \qquad \text{(B)} \end{aligned}$$

Differentiating (A) and simplifying both equations, we obtain

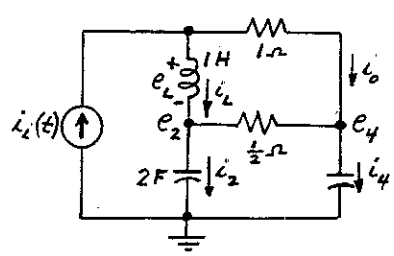
$$\begin{aligned} 2\dot{e}_A + e_A - 2\dot{e}_o &= 8\frac{di_i}{dt} \qquad \text{(C)} \\ e_A &= 2\dot{e}_o + 2e_o \qquad \text{(D)} \end{aligned}$$

Substituting (D) into (C), we get

$$\ddot{e}_o + \dot{e}_o + \frac{1}{2}e_o = 2\frac{di_i}{dt}$$

6.25

We choose e_2, e_4, and i_L as the state variables. The output equation is $i_o = i_i(t) - i_L$. Summing the currents leaving node 2, the currents leaving node 4, and the voltages around a closed path that contains the inductor and the capacitors, we have



$$\begin{aligned} i_2 + 2(e_2 - e_4) - i_L &= 0 \\ i_4 + 2(e_4 - e_2) - [i_i(t) - i_L] &= 0 \\ e_2 + e_L - e_4 - [i_i(t) - i_L] &= 0 \end{aligned}$$

We solve these equations for i_2, i_4, and e_2 and then use the element laws $\dot{e}_2 = 0.5i_2, \dot{e}_4 = 0.25i_4$, and $di_L/dt = e_L$ in order to get the state-variable equations.

$$\begin{aligned} \dot{e}_2 &= -e_2 + e_4 + \frac{1}{2}i_L \\ \dot{e}_4 &= \frac{1}{2}e_2 - \frac{1}{2}e_4 - \frac{1}{4}i_L + \frac{1}{4}i_i(t) \\ \frac{di_L}{dt} &= -e_2 + e_4 - i_L + i_i(t) \end{aligned}$$

6.30

(a) Because $i_A + i_B = i_i(t)$, we have only one state variable, which we initially choose to be i_A. By Kirchhoff's voltage law,

$$L_1 \frac{di_A}{dt} + R_1 i_A = L_2 \frac{di_B}{dt} + R_2 i_B$$

Replacing i_B by $i_i(t) - i_A$ gives

$$(L_1 + L_2)\frac{di_A}{dt} + (R_1 + R_2)i_A = L_2 \frac{di_i}{dt} + R_2 i_i(t)$$

Defining the new state variable

$$x = (L_1 + L_2)i_A - L_2 i_i(t)$$

we get

$$\dot{x} = -\left(\frac{R_1 + R_2}{L_1 + L_2}\right)x + \left(\frac{R_2 L_1 - R_1 L_2}{L_1 + L_2}\right)i_i(t)$$

where

$$i_A = \frac{1}{L_1 + L_2}[x + L_2 i_i(t)]$$

(b) The output voltage e_o is given by

$$e_o = L_1 \frac{di_A}{dt} + R_1 i_A = \frac{1}{L_1 + L_2}\left[L_1 \dot{x} + L_1 L_2 \frac{di_i}{dt} + R_1 x + R_1 L_2 i_i(t)\right]$$

Inserting the expression for $\dot{x}$ and simplifying the result, we find that

$$e_o = \frac{1}{(L_1 + L_2)^2}\left[(R_1 L_2 - R_2 L_1)q + L_1 L_2 (L_1 + L_2)\frac{di_i}{dt} + (R_2 L_1^2 + R_1 L_2^2) i_i(t)\right]$$

In this situation, we cannot avoid a derivative of the input on the right side of the output equation.

6.33

Let e_A and e_B denote the voltages at the inverting and noninverting terminals, respectively, of the op-amp. Summing the currents leaving node A gives

$$\frac{1}{R_1}e_A + \frac{1}{R_2}(e_A - e_o) + C(\dot{e}_A - \dot{e}_o) = 0 \qquad \text{(A)}$$

Because no current enters the input terminals of the op-amp, we use the voltage-divider rule to write

$$e_B = \frac{R_4}{R_3 + R_4}e_i(t) \qquad \text{(B)}$$

By the virtual-short concept, $e_B = e_A$ Using this fact, we substitute (B) into (A) to obtain

$$C\dot{e}_o + \frac{1}{R_2}e_o = \frac{R_4}{R_3 + R_4}\left[C\dot{e}_i + \frac{R_1 + R_2}{R_1 R_2}e_i(t)\right]$$