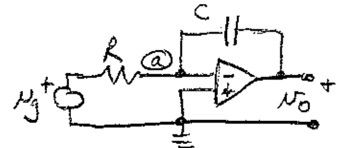
Nilsson & Riedel: 7.96, 10.7
Probs. 3 & 4 as specified

Engineering 11: Electrical Circuit Analysis
Assignment 9 Solutions

2005
LAM

1. 7.96 Assume $v_C(0) = 0$.
$C = 800\,pF$, $R = 1\,k\Omega$

$$v_g(t) = \begin{cases} \frac{2}{10^{-6}}t = 2(10^6)t & 0 \le t \le 1\mu s \\ -2(10^6)t + 4 & 1 \le t \le 3\mu s \\ 2(10^6)t - 8 & 3 \le t \le 4\mu s \end{cases}$$



$\sum_a i = 0 = \frac{v_g - 0}{R} + C\frac{d}{dt}(v_o - 0) \Rightarrow \frac{dv_o}{dt} = -\frac{1}{RC}v_g \Rightarrow v_o = -\frac{1}{RC}\int_{-\infty}^{t} v_g(t)\,dt$; $\frac{1}{RC} = \frac{1}{10^3(800)(10^{-12})} = 1.25e6$

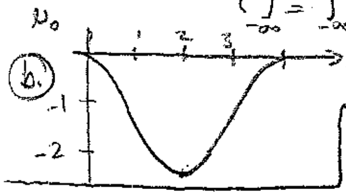
Note: $\int_{-\infty}^{t} = \int_{-\infty}^{0} + \int_{0}^{t} = \int_{0}^{t}$

So
$$v_o = 1.25(10^6)\begin{cases} -2(10^6)\frac{t^2}{2}\Big|_0^t = -10^6 t^2 \quad (= -10^{-6} @ t = 1\mu s) \\ -\left[2(10^6)\frac{t^2}{2} + 4t\right]_{1\mu s}^{t} - 10^{-6} = 10^6 t^2 - 4t - [1(10^6)(10^{-12}) - 4(10^{-6})] - 10^{-6} = 10^6 t^2 - 4t + 2(10^{-6}) \quad (= -10^{-6} @ 3\mu s) \\ -\left[2(10^6)\frac{t^2}{2} - 8t\right]_{3\mu s}^{t} - 10^{-6} = -1(10^6)t^2 + 8t - [-10^6(9)(10^{-12}) + 8(3)(10^{-6})] - 10^{-6} = -10^6 t^2 + 8t - 16(10^{-6}) \end{cases}$$

Where we used $\int_{-\infty}^{t} = \int_{-\infty}^{0} + \int_{0}^{1\mu s} + \int_{1\mu s}^{t}$ for $1 \le t \le 3\mu s$
$\int_{-\infty}^{t} = \int_{-\infty}^{0} + \int_{0}^{1\mu s} + \int_{1\mu s}^{3\mu s} + \int_{3\mu s}^{t}$ for $3 \le t \le 4\mu s$

a)
$$\boxed{v(t) = \begin{cases} -1.25(10^{12})t^2\ V & ; 0 \le t \le 1\mu s \\ 1.25(10^{12})t^2 - 5(10^6)t + 2.5\ V & 1 \le t \le 3\mu s \\ -1.25(10^{12})t^2 + 10(10^6)t - 20\ V & 3 \le t \le 5\mu s \end{cases}}$$

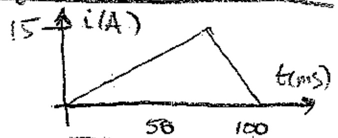
b.)



c. Yes, this repeats since $v_o = 0$ when the input begins to repeat.

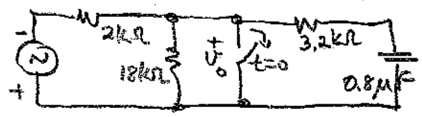
2. 10.7 Find RMS value.

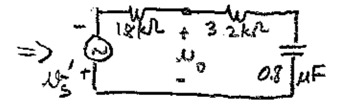
$$i(t) = \begin{cases} 200t & 0 \le t \le 75ms \\ 60 - 600t & 75 \le t \le 100ms \end{cases} \qquad I_{RMS} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$$



$$I_{RMS}^2 = \frac{1}{100m}\left\{\int_0^{75m}(200t)^2\,dt + \int_{75m}^{100m}(60 - 600t)^2\,dt\right\} = 10(5.625) + 10(1.875) = 75 \Rightarrow \boxed{I_{RMS} = \sqrt{75}\ A}$$

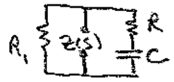
3. For a sinusoidal source, $\omega = 1000$, find $v_o(t)$.
$v_s = 40\cos(1000t)$; By source transformation, $v_s' = 36\cos(1000t)$





For $t<0$, S is closed, & $v_o = 0$. For $t>0$, $v_o = v_{oh} + v_{op}$. Use $v_C(0^-) = v_C(0^+) = 0$

Homogeneous:



$Z(s) = R_1 \| (R + \frac{1}{sC}) = R_1 \| \frac{sRC+1}{sC} = \frac{R_1(sRC+1)}{sR_1C + sRC + 1}$. Use poles for $v_{oh} \Rightarrow s = \frac{-1}{(R_1+R)C}$

$v_{oh} = Ae^{-250t}$

Particular: use voltage divider with impedances/phasors ($Z_C = \frac{1}{j\omega C}$)

$$\underline{V}_{op} = \frac{3.2k + \frac{1}{j(0.8)(10^{-3})}}{1.8k + 3.2k + \frac{1}{j(0.8)(10^{-3})}}\underline{V}_s' = \frac{3.2 - j1.2}{5 - j1.2}\underline{V}_s' = \frac{3.418\angle -20.56^\circ}{5.142\angle -13.5^\circ}(-23) = -15.3\angle 7.06^\circ$$

$\Rightarrow v_{op} = -15.3\cos(1000t - 7.06^\circ)$

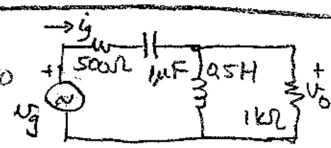
Need IC to find A. $v_C(0) = 0 \Rightarrow$

$v_o = Ae^{-250t} + 15.3\cos(1000t - 172.8^\circ)$; $v_o(0) = \frac{3.2}{3.2+1.8}(-36) \cong -23 = A + 15.3 \Rightarrow A = -38.3$

$$\therefore \boxed{v_o(t) = -38.3e^{-250t} + 15.3\cos(1000t - 172.8^\circ)\ u(t)\ V}$$

Note: other equivalent forms of this solution exist.

4. Let $v_g = 20\cos 2000t$. Insert S that closes @ $t = 0$. Assume $v_C(0^-) = i_L(0^-) = 0$
From prev week, $v_{op} = 14.14\cos(\omega t + \pi/4)$. Find v_{oh} using $Z(s)$.



$$Y(s) = \frac{1}{R_1} + \frac{1}{sL} + \frac{1}{R + 1/sC} = \frac{s^2(R+R_1)LC + s(RR_1C + L) + R_1}{sR_1L(sRC+1)}$$; use zeros of Y for v_{oh}

$$s^2 + s\frac{RR_1C + L}{(R+R_1)LC} + \frac{R}{(R+R_1)LC} = 0$$; solve for s using above values.

Finished ICs in class.