9.2, 9.32, 9.40, 9.41, 9.63, 9.64

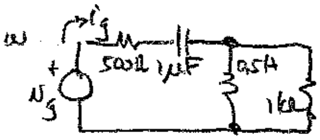
Engineering 11
Problem Set 8 Solutions

2005
LAM

(1. 9.2) Find V_{RMS} for $v = V_m \sin \frac{2\pi}{T} t \quad 0 \le t \le T/2 \qquad V_{RMS}^2 = \frac{1}{T}\left[\int_0^{T/2} V_m^2 \sin^2 \frac{2\pi}{T} t\, dt + \int_{T/2}^{T} 0\, dt\right]$

$$V_{RMS}^2 = \frac{V_m^2}{2T}\int_0^{T/2}\left(1-\cos\frac{4\pi}{T}t\right)dt = \frac{V_m^2}{2T}\left[\int_0^{T/2} dt - \int_0^{T/2}\cos\frac{4\pi}{T}t\, dt\right] = \frac{V_m^2}{4} \Rightarrow \boxed{V_{RMS} = \frac{V_m}{2}}$$

(2. 9.32) a. Freq of source adjusted to give i_g in phase of v_g. Find ω



Want $Z(s)$ to be purely real:

$$Z(s) = R_1 + \frac{1}{sC} + \frac{sLR_2}{sL+R_2} = \frac{sR_1C+1}{sC} + \frac{sLR_2}{sL+R_2}$$
$$= \frac{s^2(R_1+R_2)LC + s(L+R_1R_2C) + R_2}{sC(sL+R_2)}$$

let $s \to j\omega \Rightarrow Z_{(j\omega)} = \dfrac{\overbrace{R_2-\omega^2(R_1+R_2)LC}^{a_1} + j\overbrace{\omega(L+R_1R_2C)}^{b_1}}{-\omega^2LC + j\omega R_2C} \cdot \dfrac{\overbrace{-\omega^2LC}^{a_2} - \overbrace{j\omega R_2C}^{b_2}}{-\omega^2LC - j\omega R_2C}$ makes denominator purely real.

Numerator in the form $(a_1+jb_1)(a_2+jb_2)$; want that to be purely real as well $\Rightarrow$

$j(a_1b_2 + a_2b_1) = 0$ or $(R_2-\omega^2(R_1+R_2)LC)(-\cancel{\omega}R_2C) + (-\omega^2LC)(\cancel{\omega})(L+R_1R_2C) = 0$

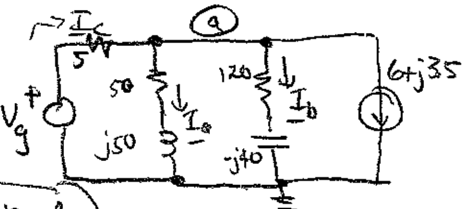
plug in numbers & obtain quadratic eq; solve to find $\boxed{\omega = 2000 \text{ rad/s}}$

b. If $v_g = 20\cos 2000t$ V, find $v_o \rightarrow Z(j2000) = 500 - j500 + (j1000 \| 1000) = 1000\,\Omega$

$i_g = 20\cos 2000t$ mA $\Leftarrow$ $\underline{I}_g = 20\angle 0$ mA, $\underline{V}_g = 20\angle 0$ V

$\boxed{v_o = 10\sqrt{2}\cos(2000t + \pi/4) \text{ mA}}$ $\Leftarrow$ voltage ÷ $\Rightarrow \underline{V}_o = \dfrac{(j1000 \| 1000)}{500 - j500 + (j1000\|1000)}\underline{V}_g = 10\sqrt{2}\angle \pi/4$

(3. 9.40) $\underline{I}_a = 2\angle 0 \Rightarrow \underline{V}_a = (50+j150)(2\angle 0) = 100 + j300$
$= 316\angle 1.25 \text{ rad}$



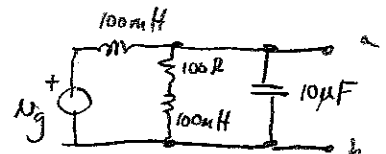
a. Find $\underline{I}_b$, $\underline{I}_c$, $\underline{V}_g$

$$\underline{I}_b = \frac{316\angle 1.25 \text{ rad}}{120 - j40 = 126\angle -.32 \text{ rad}} = \boxed{2.5\angle \pi/2}$$

$\underline{I}_c = \underline{I}_a + \underline{I}_b + \underline{I}_s = 2 + j2.5 + 6 + j35 = 8 + j6 = \boxed{10\angle .64 \text{ rad}}$

$\underline{V}_g = 5\underline{I}_c + \underline{V}_a = 5(8+j6) + 100 + j300 = 140 + j330 \cong \boxed{358\angle 1.17 \text{ rad}}$

b. $\omega = 800$ rad/s $\Rightarrow$ $i_b(t) = 2.5\cos(800t + \pi/2)$ A
$i_c(t) = 10\cos(800t + 0.64)$ A
$v_g(t) = 358\cos(800t + 1.17)$ V



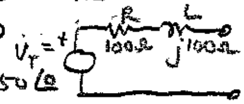
$Z_L = j\omega L = j100\,\Omega$
$Z_C = -j\frac{1}{\omega C} = -j100\,\Omega$
$Z_{RL} = 100 + j100\,\Omega$

(4. 9.41) $v_g = 247.49\cos(1000t + \pi/4)$ V

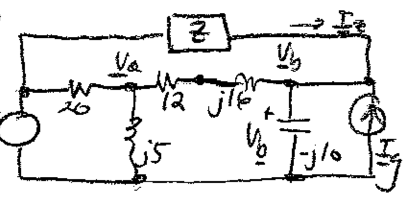
a. Find $V_T = \dfrac{Z_C \| Z_{RL}}{Z_L + Z_C \| Z_{RL}} V_g = \dfrac{\frac{(-j100)(100+j100)}{100}}{j100 + \frac{(-j100)(100+j100)}{100}}(247.49)\angle \pi/4 = 350\angle 0°$

b. Find Z_T (remove sources & look in) $\Rightarrow Z_T = Z_L \| Z_{RL} \| Z_C$ or $Y_T = \frac{1}{Z_T} = \frac{1}{Z_L} + \frac{1}{Z_{RL}} + \frac{1}{Z_C}$

$Y_T = \frac{1}{j100} + \frac{1}{100+j100} + \frac{1}{-j100} = \frac{1}{100\sqrt{2}\angle \pi/4} \Rightarrow Z_T = 100\sqrt{2}\angle \pi/4 = 100 + j100$



(5. 9.63) Find Z if $\underline{V}_g = 100 - j50$, $\underline{I}_g = 30 + j20$, & $\underline{V}_b = 140 + j30$



$$\sum_a \underline{I} = \frac{\underline{V}_g - \underline{V}_a}{20} - \frac{\underline{V}_a}{j5} - \frac{\underline{V}_a - \underline{V}_b}{12+j16} = 0 \Rightarrow \frac{100 - j50 - \underline{V}_a}{20} - \frac{\underline{V}_a}{j5} - \frac{\underline{V}_a - 140 - j30}{12+j16} = 0$$

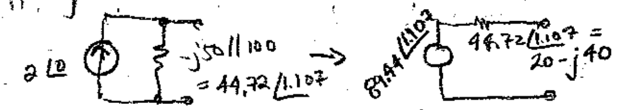
$$\sum_b \underline{I} = \frac{\underline{V}_a - \underline{V}_b}{12+j16} - \frac{\underline{V}_b}{-j10} + \underline{I}_g + \underline{I}_z = 0 = \frac{\underline{V}_a - 140 - j30}{12+j16} - \frac{140+j30}{-j10} + 30 + j20 - \underline{I}_z$$

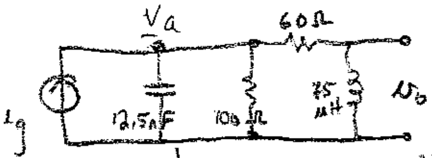
Solve $\sum_a \underline{I}$ for $\underline{V}_a = 40 + j30$ & $\sum_b \underline{I}$ for $\underline{I}_z = -30 - j10$

$$\& \; Z = \frac{\underline{V}_g - \underline{V}_b}{\underline{I}_z} = \frac{(100 - j50) - (140 + j30)}{-30 - j10} = \boxed{2 + j2\ \Omega}$$

6. 9.64 a) Find v_o if $i_g = 2\cos(16\cdot 10^5 t)$ A $\Rightarrow \underline{I}_g = 2\angle 0$

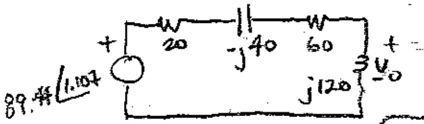
Source transformation





$$Z_C = \frac{1}{j(16)(10^5)(12.5)(10^{-9})} = -j50$$

$$Z_L = j(16)(10^5)(75)(10^{-6}) = j120$$



$$\underline{V}_o = \frac{j120}{80 + j80}(89.44)\underset{(\text{rad})}{\angle 1.107} = 94.87\underset{(\text{degrees})}{\angle -18.43^\circ}$$

$v_o = 94.87\cos(16\cdot 10^5 t - 18.43^\circ)$ V

b.) Find time lag $T = \frac{2\pi}{\omega} = 1.25\pi\ \mu s$ so $\frac{18.43^\circ}{360^\circ}(1.25\pi) = \boxed{201\ ns;\ v_o \text{ lags } i_g.}$